Exercises
Session 6 (chapter 5)

1 Photoplethysmography

Let us consider a wrist photoplethysmography sensor (see Figure 1). We have a light emitter with light intensity I and a detector that picks up the light intensity i. This exercise will familiarize you with the Beer-Lambert law and its applicability in heterogeneous tissues.

Figure 1: Wrist photoplethysmography

The emitted light is split into several paths. Only some of them reach the detector. The intensity i is the sum of the contributions of all arriving paths k:

$$i = \sum_k i_k$$

1.1. Same material for the whole path

<u>Exercise statement</u>

Express the light intensity i_1 at the output of path 1. Assume a homogeneous medium with absorption coefficient α_1 (mm^{-1}) along the entire path of length l_1 as shown in Figure 2.

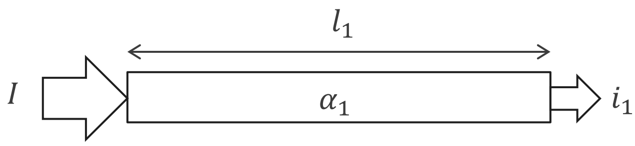

Figure 2: Homogeneous medium with absorption coefficient α_1 (mm^{-1})

1.2. Pulsatile component

The light propagating along path 2 is assumed to cross different tissues, namely skin α_{21}, muscle α_{22}, and blood α_{23} (see Figure 3). During systole, the arterioles are slightly dilated and more light is absorbed (see Figure 4). The additional absorption is modeled by the length l_{24}, the molar absorption coefficient is ε (mm⁻¹/M) and the molar concentration is c (M). During diastole, the length l_{24} is zero.

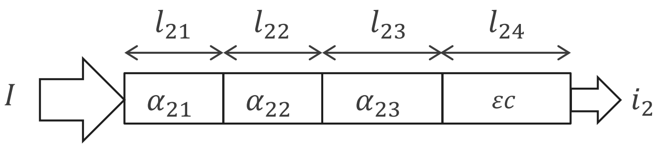

Figure 3: Path with heterogenous media

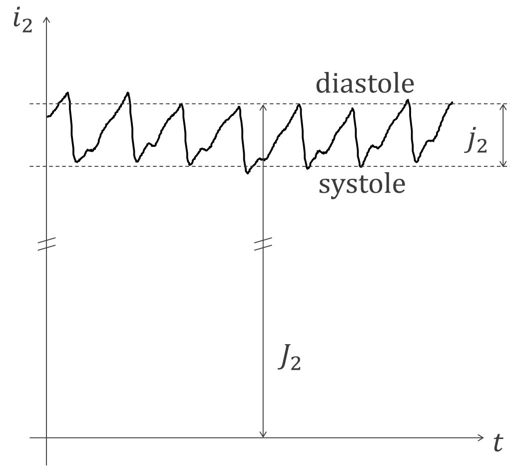

Figure 4: Signal i_2 modulated by systole/diastole cycle

Exercise statement

Express the light intensity i_2 at the output of path 2, during diastole and during systole.

1.3. Approximation

Since the pulsatile component is small (j_2 is small compared to J_2 as shown in Figure 4), a Taylor expansion can be performed.

Exercise statement

Express the light intensity at the detector during systole for path 2, i.e., i_2(systole), using only the first two terms of the Taylor expansion.

1.4. Perfusion

Perfusion P_2 is defined as the ratio of j_2 to J_2 (see Figure 4).

Exercise statement

Express the perfusion P_2 for path 2.

1.5. Varying concentration

Suppose that the concentration c is no longer constant along path 3, i.e., c is redefined as the product $cC(x)$ where the new c is a constant and $C(x)$ is a unitless function that models the variation of the concentration along x. For the sake of the exercise, assume that the function $C(x)$ is (see Figure 5):

$$C(x) = \sin\frac{\pi x}{l_{32}}$$

i.e., that the concentration $cC(x)$ is 0 at $x = 0$ and at $x = l_{32}$ with a maximum at c in between.

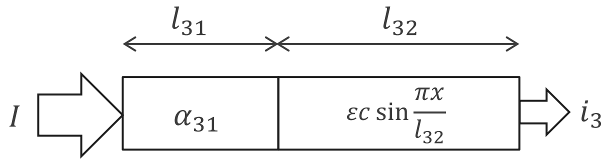

Figure 5: Path with medium of varying concentration

<u>Exercise statement</u>

Express the light intensity i_3 at the output of path 3 during systole.

1.6. Total intensity

The light intensity i arriving at the detector in our exercise is:

$$i = i_1 + i_2 + i_3$$

Therefore, we also have:

$$J = J_1 + J_2 + J_3, \qquad j = j_1 + j_2 + j_3$$

<u>Exercise statement</u>

Express J and j and calculate L for the perfusion to be:

$$P = \frac{j}{J} = \varepsilon c L$$